OCR RECOGNISING ACHIEVEMENT

Friday 18 May 2012 – Morning

AS GCE MATHEMATICS

4722 Core Mathematics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4722
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

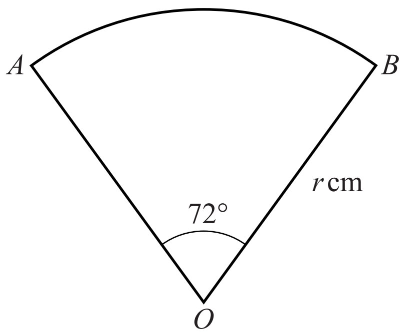
1 **(i)** Find the binomial expansion of $(3 + 2x)^5$, simplifying the terms. **[4]**

(ii) Hence find the binomial expansion of $(3 + 2x)^5 + (3 - 2x)^5$. **[2]**

2 **(i)** Find $\int (x^2 - 2x + 5)\,dx$. **[3]**

(ii) Hence find the equation of the curve for which $\frac{dy}{dx} = x^2 - 2x + 5$ and which passes through the point (3, 11). **[3]**

3



The diagram shows a sector *AOB* of a circle, centre *O* and radius *r* cm. Angle *AOB* is 72°.

(i) Express 72° exactly in radians, simplifying your answer. **[1]**

The area of the sector *AOB* is 45π cm^2.

(ii) Find the value of *r*. **[2]**

(iii) Find the area of the segment bounded by the arc *AB* and the chord *AB*, giving your answer correct to 3 significant figures. **[3]**

4 Solve the equation

$$4\cos^2 x + 7\sin x - 7 = 0,$$

giving all values of *x* between 0° and 360°. **[6]**

5 **(a)** A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 4 \quad \text{and} \quad u_{n+1} = \frac{2}{u_n} \quad \text{for } n \geqslant 1.$$

(i) Write down the values of u_2 and u_3. **[2]**

(ii) Describe the behaviour of the sequence. **[1]**

(b) In an arithmetic progression the ninth term is 18 and the sum of the first nine terms is 72. Find the first term and the common difference. **[5]**

6 **(i)** Use the trapezium rule, with 2 strips each of width 4, to show that an approximate value of $\int_1^9 4\sqrt{x}\,dx$ is $32 + 16\sqrt{5}$. **[3]**

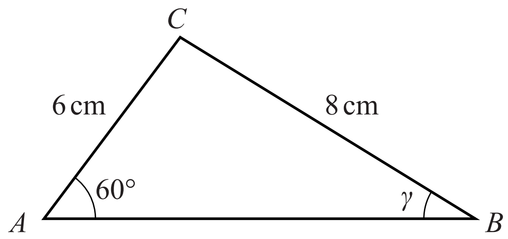
(ii) Use a sketch graph to explain why the actual value of $\int_1^9 4\sqrt{x}\,dx$ is greater than $32 + 16\sqrt{5}$. **[2]**

(iii) Use integration to find the exact value of $\int_1^9 4\sqrt{x}\,dx$. **[4]**

7 **(a)** **(i)** Given that α is the acute angle such that $\tan\alpha = \frac{2}{5}$, find the exact value of $\cos\alpha$. **[2]**

(ii) Given that β is the obtuse angle such that $\sin\beta = \frac{3}{7}$, find the exact value of $\cos\beta$. **[3]**

(b)



The diagram shows a triangle ABC with AC = 6 cm, BC = 8 cm, angle BAC = 60° and angle $ABC = \gamma$. Find the exact value of $\sin\gamma$, simplifying your answer. **[3]**

8 Two cubic polynomials are defined by

$$f(x) = x^3 + (a - 3)x + 2b, \quad g(x) = 3x^3 + x^2 + 5ax + 4b,$$

where a and b are constants.

(i) Given that f(x) and g(x) have a common factor of $(x - 2)$, show that $a = -4$ and find the value of b. **[6]**

(ii) Using these values of a and b, factorise f(x) fully. Hence show that f(x) and g(x) have two common factors. **[5]**

9 **(a)** An arithmetic progression has first term $\log_2 27$ and common difference $\log_2 x$.

(i) Show that the fourth term can be written as $\log_2 (27x^3)$. **[3]**

(ii) Given that the fourth term is 6, find the exact value of x. **[2]**

(b) A geometric progression has first term $\log_2 27$ and common ratio $\log_2 y$.

(i) Find the set of values of y for which the geometric progression has a sum to infinity. **[2]**

(ii) Find the exact value of y for which the sum to infinity of the geometric progression is 3. **[5]**

Question			Answer	Marks	Guidance	
1	(i)		$(3 + 2x)^5 = 243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$	M1*	Attempt expansion – products of powers of 3 and $2x$	Must attempt at least 5 terms. Each term must be an attempt at a product, including binomial coeffs if used. Allow M1 for no, or incorrect, binomial coeffs. Powers of 3 and $2x$ must be intended to sum to 5 within each term (allow slips if intention correct). Allow M1 even if powers used incorrectly with the $2x$ ie $2x^3$ not $(2x)^3$ can get M1. Allow M1 for powers of ${}^2/_3x$ from expanding $k(1 + {}^2/_3x)^5$, any k (allow if powers only applied to x and not ${}^2/_3$) .
				M1d*	Attempt to use correct binomial coefficients	At least 5 correct from 1, 5, 10, 10, 5, 1 - allow missing or incorrect (but not if raised to a power). May be implied rather than explicit. Must be numerical eg ${}^5C_1$ is not enough. They must be part of a product within each term. The coefficient must be used in an attempt at the relevant term ie 5 x 3^3 x $(2x)^2$ is M0. Allow M1 for correct coefficients from expanding $k(1 + {}^2/_3x)^5$, any k.
				A1	Obtain at least four correct simplified terms	Either linked by '+' or as part of a list.
				A1	Obtain fully correct expansion	With all coefficients simplified. Terms must be linked by '+' and not just commas.
				[4]		**SR for reasonable expansion attempt:** M2 for attempt involving all 5 brackets resulting in a quintic with at most one term missing A1 for four correct, simplified, terms A1 for fully correct, simplified, expansion

Question			Answer	Marks	Guidance	
1	(ii)		$(3 + 2x)^5 + (3 - 2x)^5$ $= 486 + 2160x^2 + 480x^4$	M1	Attempt to change signs of relevant terms	Must change the sign on all of the relevant terms from their expansion, and no others. Expansion in part (i) must have at least 5 terms. Allow M1 even if no attempt to then combine expansions, or if difference rather than sum found. If expanding $(3 - 2x)^5$, then it must be a reasonable attempt, involving the product of correct binomial coeffs, powers of 2 and powers of $-3x$, and each term must be of the correct sign.
				A1 FT **[2]**	Obtain $486 + 2160x^2 + 480x^4$, from their **(i)**	Must have been a 6 term quintic in (i) to get FT mark. A0 if subsequent division by a common factor, so not isw.

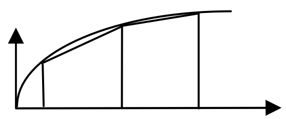
Question			Answer	Marks	Guidance	
2	(i)		$\int(x^2-2x+5)\mathrm{d}x = \frac{1}{3}x^3 - x^2 + 5x + c$	M1	Attempt integration	An increase in power by 1for at least 2 terms. Allow if the +5 disappears.
				A1	Obtain two correct (algebraic) terms	Allow if the coefficient if x^2 isn't yet simplified.
				A1	Obtain fully correct expression (allow no + c)	Allow if the coefficient if x^2 isn't yet simplified. A0 if integral sign or dx still present in their answer (but allow $\int$ = ...). A0 if a list of terms rather than an expression.
				[3]		
2	(ii)		$y = \frac{1}{3}x^3 - x^2 + 5x + c$	M1*	State or imply y = their integral from (i)	Must have come from integration attempt ie the M1 must have been gained in part (i). Allow slips when transferring expression from (i). Can still get this M1 if no + c. The y does not have to be explicit - it could be implied by eg 11 = F(3) (but not by 3 = F(11)). Using definite integration with limits of 3 & 11 is M0. M0 if they start with y = their integral from (i), but then attempt to use $y - 11 = m(x - 3)$. This is a re-start and gains no credit.
			$11 = 9 - 9 + 15 + c \Rightarrow c = -4$	M1d*	Attempt to find c using (3, 11)	Need to get as far as attempting c. M1 could be implied by eg 11 = 9 – 9 + 15 and then an attempt to include a constant to balance the eqn, even though + c never actually seen. M0 if no + c seen or implied. M0 if using $x = 11$, $y = 3$.
			hence $y = \frac{1}{3}x^3 - x^2 + 5x - 4$	A1	Obtain $y = \frac{1}{3}x^3 - x^2 + 5x - 4$	Coeff of x^2 now needs to be simplified (A0 for $-1x^2$). Must be an equation ie y = ..., so A0 for 'f(x) = ...' or 'equation = ...' Allow aef, such as $3y = x^3 - 3x^2 + 15x - 12$.
				[3]		

Question			Answer	Marks	Guidance	
3	(i)		$72^o = 72 \times {}^{\pi}/_{180} = {}^{2\pi}/_5$ radians	B1 **[1]**	State $\frac{2\pi}{5}$, $\frac{2}{5}\pi$ or 0.4π	Must be simplified. B0 for decimal equiv (1.26), but isw if exact simplified value seen but then given in decimals.
3	(ii)		$½ \times r^2 \times {}^{2\pi}/_5 = 45\pi$	M1	Equate attempt at area using $(½)r^2\theta$ to 45π and attempt to solve for r	Condone omission of ½, but no other error. Must be equated to 45π (so M0 for = 45) except for $(½)r^2$ x ${}^2/_5 = 45$ (assuming π has been cancelled). Allow M1 for using 0.4 or 1.26π. Allow if using incorrect angle from part (i), as long as clearly intended to be in radians. Allow equivalent method using fractions of the circle. Valid method using degrees can get M1 (and A1 if exact). Must get as far as attempt at r. Allow M1 for T&I, as long as comparing to 45π.
			$r = 15$ cm	A1 **[2]**	Obtain $r = 15$	Must be exact working only – any use of decimals is A0 (but allow 15.0 if giving previously exact answer to 3sf). Any evidence of 15 having come from rounding an inaccurate answer is A0.
3	(iii)		area triangle = $½ \times 15^2 \times \sin({}^{2\pi}/_5) = 106.99$	M1*	Attempt area of triangle using $(½)r^2\sin\theta$	Condone omission of ½, but no other error. Must be using their r and angle linked to their θ. Could be using degrees or radians. Allow even if evaluated in incorrect mode (deg mode gives 2.47, rad mode gives 28.66). If using a right-angled triangle, it must be $½bh$, any valid use of trig to find b and h.
			area segment = $45\pi - 106.99$	M1d*	Attempt 45π – area of triangle	Must be using 45π (not 45), decimal equiv of 141.4 or ½ x 15^2 x 1.26 (or better), so M0 for any other value. M0 if area of triangle is greater than 45π. Using ½ x 15^2 x (θ - sin θ) with incorrect θ is M1M0.
			$= 34.4$ cm^2	A1 **[3]**	Obtain 34.4 cm^2	If >3sf then allow any values rounding to 34.4.

Question			Answer	Marks	Guidance	
4			$4(1-\sin^2 x) + 7\sin x - 7 = 0$	M1	Use $\cos^2 x = 1 - \sin^2 x$	Must be used and not just stated Must be used correctly, so M0 for $1 - 4\sin^2 x$.
			$4\sin^2 x - 7\sin x + 3 = 0$	A1	Obtain correct quadratic	aef, as long as three term quadratic with all the terms on one side of the equation. Condone $4\sin^2 x - 7\sin x + 3$ ie no = 0.
			$(\sin x - 1)(4\sin x - 3) = 0$	M1	Attempt to solve quadratic in $\sin x$	Not dependent on previous M1, so could get M0M1 if $\cos^2 x = \sin^2 x - 1$ used. This M mark is just for solving a 3 term quadratic (see guidance sheet for acceptable methods). Condone any substitution used, inc $x = \sin x$.
			$\sin x = 1$, $\sin x = {}^3/_4$	M1	Attempt to find x from roots of quadratic	Attempt $\sin^{-1}$ of at least one of their roots. Allow for just stating $\sin^{-1}$ (their root) inc if $\lvert\sin x\rvert > 1$. Not dependent on previous marks so M0M0M1 poss. If going straight from $\sin x = k$ to $x = \ldots$, then award M1 only if their angle is consistent with their k.
			$x = 90°$ $x = 48.6°, 131°$	A1	Obtain two correct solutions	Allow 3sf or better. Must come from a correct solution of the correct quadratic – if the second bracket was correct but the first was $(\sin x + 1)$ then A0 even though 2 solutions will be as required. Allow radian equivs – ${}^{\pi}/_2$ or 1.57 / 0.848 / 2.29.
				A1	Obtain all 3 correct solutions, and no others	Must now all be in degrees. Allow 3sf or better. A0 if other incorrect solutions in range $0^o - 360^o$ (but ignore any outside this range).
				[6]		**SR** If no working shown then allow **B1** for each correct solution (max of **B2** if in radians, or if extra solns in range).

Question			Answer	Marks	Guidance	
5	(a)	(i)	$u_2 = ½$	B1	State ½	Allow 0.5 or $^2/_4$.
			$u_3 = 4$	B1 FT	State 4, following their u_2	Follow through on their u_2 (simplifying if possible). B0 for $^2/_{0.5}$, $^2/_{½}$ etc.
				[2]		
5	(a)	(ii)	periodic / alternating / repeating / oscillating / cyclic	B1	Any correct description	Allow associated words eg 'repetitive'. Must be a mathematical term rather than a description such as 'it changes between 4 and ½' or 'odd terms are 4, even terms are ½. Mark independently of any values given in part (i). Ignore irrelevant terms (eg 'recursive'), but B0 if additional incorrect terms (eg 'geometric').
				[1]		
5	(b)		$a + 8d = 18$	B1	State $a + 8d = 18$	Allow any equivalent, including unsimplified. Must be correct when seen – can't be implied by eg being stated but with incorrect a substituted.
			$^9/_2\ (2a + 8d) = 72$	B1	State $^9/_2\ (2a + 8d) = 72$	Allow any equivalent, including unsimplified. Must be correct when seen – as above.
			$a + 8d = 18$ and $2a + 8d = 16$	M1	Attempt to solve simultaneously	M1 is awarded for eliminating a variable from two linear equations in a and d, from attempt at $u_9 = 18$ and attempt at $S_9 = 72$ (formulas must be recognisable, and for APs, but not necessarily correct). Don't need to actually solve. If balancing equations, then there must be intention to subtract (but allow $a = 2$). If substituting then allow sign errors (eg $a = 8d – 18$), but not operational errors (eg $a = {}^{18}/_{8d}$).
			$a = –2$, $d = {}^5/_2$	A1	Obtain either $a = –2$ or $d = {}^5/_2$	A1 is given for the first correct value, from 2 correct eqns. Allow $d = 2½$ or 2.5, but not unsimplified fractions.
				A1	Obtain both $a = –2$, $d = {}^5/_2$	A1 is given for obtaining second correct value. Allow $d = 2½$ or 2.5, but not unsimplified fractions.
				[5]		

Question			Answer	Marks	Guidance	
			OR		**Alternative method using $^n/_2(a + l)$**	**NB If using $a + (n - 1)d = 18$ and $^9/_2\ (2a + (n - 1)d) = 72$ and solving simultaneously to get a and d, then mark as per scheme below.**
			$^9/_2\ (a + 18) = 72$	B1*	State $^9/_2\ (a + 18) = 72$	Allow any equivalent. Award B1 as soon as seen correct, even if subsequent error.
			$a = -2$	B1d*	Obtain $a = -2$	Must come from correct equation.
			$-2 + 8d = 18$ or $^9/_2\ (-4 + 8d) = 72$	M1	Attempt use of either u_9 or S_9	Must be attempting either $u_9 = 18$ or $S_9 = 72$. Must be using correct formula.
				A1FT	Obtain correct equation, following their a	Allow any equivalent, including unsimplified.
			$d = {}^5/_2$	A1	Obtain $d = {}^5/_2$	Allow $d = 2\frac{1}{2}$ or 2.5, but not unsimplified fractions.

Question			Answer	Marks	Guidance	
6	(i)		$0.5 \times 4 \times (4\sqrt{1} + 8\sqrt{5} + 4\sqrt{9})$	M1*	Attempt y-values at $x = 1, 5, 9$ only	Must be using y, not an attempt at integration. Allow slips eg $\sqrt{(4x)}$ as long as clearly intended as y. Allow decimal equiv for y_1 (8.94). Allow M1 for 4, 20, 72 (ie omitting the √). M0 if other y-values found (unless not used in trap rule).
			$= 2(16 + 8\sqrt{5})$	M1d*	Attempt correct trapezium rule, inc $h = 4$	Correct structure, including 'big brackets' seen or implied. Allow 2 used for ½ h – no need for ½ x 4 to be explicit. Allow slips when calculating y values, but all other aspects must be correct. Could use two separate trapezia.
			$= 32 + 16\sqrt{5}$ **AG**	A1 **[3]**	Obtain $32 + 16\sqrt{5}$	Must come from exact working, so A0 if answer first found in decimals (67.777…) which is then stated to be the same as $32 + 16\sqrt{5}$. However, isw if exact answer found first, and then decimal equiv stated.
6	(ii)			B1*	Sketch showing correct graph of $y = 4\sqrt{x}$ and two trapezia (allow if only tops of trapezia seen as chords)	Correct graph shown, existing for at least $1 \le x \le 9$. Exactly two trapezia must be shown, of roughly equal widths, with top vertices on the curve.
			Curve is above tops of trapezia	B1d* **[2]**	Reason comparing the tops of trapezia to the curve, or referring to the gap between the trapezia and the curve	Must refer to the tops of the trapezia so B0 for 'trapezia are below curve' (ie 'top' not used). Allow 'trapezium' rather than 'trapezia'. Could shade gaps on their diagram but some text also reqd. B0 for 'some area not calculated' unless clear which area. Concave / convex is B0, as is comparing to exact area. B1 for decreasing gradient (but B0 for decreasing curve). B0 (rather than isw) if explanation is partially incorrect. No sketch is B0, irrespective of explanation given. **SR B1** for correct explanation, and trapezia, and correct graph of $y = 4\sqrt{x}$ for $1 \le x \le 9$ but incorrect outside range (eg curvature / y-intercept / not just in first quadrant).

Question			Answer	Marks	Guidance	
6	(iii)		$\int_1^9 4x^{\frac{1}{2}}\,dx = \left[\frac{8}{3}x^{\frac{3}{2}}\right]_1^9$	M1	Obtain $k\,x^{\frac{3}{2}}$	Any numerical k, including 4. Any exact equiv for the index.
			$= 72 - \frac{8}{3}$	A1	Obtain $\frac{8}{3}x^{\frac{3}{2}}$	Allow unsimplified coefficient, inc $\frac{4}{1.5}$ or $\frac{2}{3}\times 4$. Allow non exact decimal ie 2.7, 2.67 etc. Allow + c.
			$= 69^1/_3$	M1	Attempt correct use of limits	Must be F(9) – F(1) ie subtraction with limits in the correct order. Allow use in any function other than the original, including from differentiation. Allow processing errors eg $\left(\frac{8}{3}\times 9\right)^{1.5}$.
				A1	Obtain $69^1/_3$, or any exact equiv	Allow improper fraction, or recurring decimal. A0 for 69.333…. A0 for $69^1/_3 + c$.
				[4]		Answer only is 0/4.

Question			Answer	Marks	Guidance	
7	(a)	(i)	$\cos\alpha = {}^5/_{\sqrt{29}}$	M1	Attempt cos α	Could draw triangle and use Pythagoras to find the hypotenuse, or use trig identities. Must get as far as attempting cos α. Must be working in exact values for M1. Must be using correct ratios for tan α and cos α.
				A1	Obtain ${}^5/_{\sqrt{29}}$	Allow any exact equiv, including rationalised surd or $\sqrt{({}^{25}/_{29})}$ isw if decimal equiv subsequently given. Answer only gets full credit.
				[2]		**SR B1** for exact answer following decimal working.
7	(a)	(ii)	$\cos\beta = {}^{-\sqrt{40}}/_7$	M1	Attempt cos *β*	Could draw triangle and use Pythagoras to find the adjacent, or use trig identities. Must get as far as attempting cos *β*. Must be working in exact values for M1. Must be using correct ratios for sin α and cos α.
				A1	Obtain ${}^{\sqrt{40}}/_7$	Allow any exact equiv, including $\sqrt{({}^{40}/_{49})}$. Allow ± ${}^{\sqrt{40}}/_7$ (from using $\cos^2 x = 1 - \sin^2 x$). isw if decimal equiv subsequently given. Answer only gets M1A1.
				A1 FT	Obtain ${}^{-\sqrt{40}}/_7$, or –ve of their exact numerical value for cos *β*	A1 FT can only be awarded following M1. isw if decimal equiv subsequently given. Answer only gets full credit.
				[3]		**SR B1** for ${}^{\sqrt{40}}/_7$, or equiv, following decimal working **SR B2** for ${}^{-\sqrt{40}}/_7$, or equiv, following decimal working **SR B1** for decimal answer in range [-0.904, -0.903]

Question			Answer	Marks	Guidance	
7	(b)		$\frac{\sin\gamma}{6}=\frac{\sin 60}{8}$	M1*	Attempt use of correct sine rule	Must be correct sine rule, either way up (just need to substitute values in – no rearrangement needed).
				M1d*	Use $\sin 60° = \sqrt{3}/2$	Could be implied eg $\frac{6}{\sin\gamma}=\frac{16}{3}\sqrt{3}$
			$\sin\gamma = \frac{3\sqrt{3}}{8}$	A1	Obtain $\sin\gamma$ as $\frac{3\sqrt{3}}{8}$	Must be seen simplified to this, or $0.375\sqrt{3}$ or $\frac{9}{8\sqrt{3}}$, but isw if decimal equiv subsequently given. isw any attempt to find the angle. A0 if only ever seen as $\sin^{-1}\frac{3\sqrt{3}}{8}$
				[3]		

Question			Answer	Marks	Guidance	
8	(i)		$f(2) = 8 + 2a - 6 + 2b = 0$ $g(2) = 24 + 4 + 10a + 4b = 0$	M1	Attempt at least one of f(2), g(2)	Allow for substituting $x = 2$ into either equation – no need to simplify at this stage. Division – complete attempt to divide by $(x - 2)$. Coeff matching - attempt all 3 coeffs of quadratic factor.
				M1	Equate at least one of f(2) and g(2) to 0	Just need to equate their substitution attempt to 0 (but just writing eg f(2) = 0 is not enough). It could be implied by later working, even after attempt to solve equations. Division - equating their remainder to 0. Coeff matching – equate constant terms.
			$2a + 2b = -2$, $5a + 2b = -14$	A1	Obtain two correct equations in a and b	Could be unsimplified equations. Could be $8a + 2b = -26$ (from f(2) = g(2)).
			hence $3a = -12$	M1	Attempt to find a (or b) from two simultaneous eqns	Equations must come from attempts at two of f(2) = 0, g(2) = 0, f(2) = g(2). M1 is awarded for eliminating a or b from 2 sim eqns – allow sign slips only. Most will attempt a first, but they can also gain M1 for finding b from their simultaneous equations.
			so $a = -4$ **AG**	A1	Obtain $a = -4$, with necessary working shown	If finding b first, then must show at least one line of working to find a (unless earlier shown explicitly eg $a = -1 - b$).
			$b = 3$	A1	Obtain $b = 3$	Correct working only
				[6]		**SR Assuming *a* = -4** **Either use this scheme, or the original, but don't mix elements from both** M1 Attempt either f(2) or g(2) M1 Equate f(2) or g(2) to 0 (also allow f(2) = g(2)) A1 Obtain $b = 3$ A1 Use second equation to confirm $a = -4$, $b = 3$

Question			Answer	Marks	Guidance	
8	(ii)		$f(x) = (x-2)(x^2+2x-3)$ $= (x-2)(x+3)(x-1)$	M1	Attempt full division of their $f(x)$ by $(x-2)$ Could also be for full division attempt by $(x-1)$ or $(x+3)$ if identified as factors	Must be using $f(x) = x^3 - 7x + k$. Must be complete method – ie all 3 terms attempted. Long division – must subtract lower line (allow one slip). Inspection – expansion must give at least three correct terms of their cubic. Coefficient matching – must be valid attempt at all 3 quadratic coeffs, considering all relevant terms each time. Factor theorem – must be finding 2 more factors / roots.
				A1	Obtain x^2 and at least one other correct term, from correct $f(x)$	Could be middle or final term depending on method. Coeff matching – allow for stating values eg $A = 1$ etc. Factor theorem – state factors of $(x+3)$ and $(x-1)$.
				A1	Obtain $(x-2)(x+3)(x-1)$	Must be seen as a product of three linear factors. Answer only gains all 3 marks.
			$g(x) = (x-2)(3x^2+7x-6)$ $= (x-2)(x+3)(3x-2)$ **OR** $g(1) = -4$, $g(-3) = 0$	M1	Attempt to verify two common factors	Possible methods are: Factorise $g(x)$ completely – $f(x)$ must have been factorised. Find quadratic factor of $g(x)$ and identify $x = -3$ as root. Test their roots of $f(x)$ in $g(x)$. Just stating eg $g(-3) = 0$ is not enough – working required. If $f(x)$ hasn't been factorised, allow M1 for using factor thm on both functions to find common factor, or for factorising $g(x)$ and testing roots in $f(x)$.
			Hence common factor of $(x+3)$	A1 **[5]**	Identify $(x+3)$ as a common factor	Just need to identify $(x+3)$ - no need to see $(x-2)$ or to explicitly state 'two common factors'. Need to see $(x+3)$ as factor of $g(x)$ – just showing $g(-3) = 0$ and then stating 'common factor' is not enough. CWO (inc A0 for $g(x) = (x-2)(x+3)(x-{}^{2}/_{3})$). If using factor thm, no need to find $g(1)$ if $g(-3)$ done first. Just stating $(x+3)$ with no supporting evidence is M0A0. A0 if referring to -3 (and 2) as 'factors'. A0 if additional incorrect factor given.

Question			Answer	Marks	Guidance	
9	(a)	(i)	$u_4 = \log_2 27 + 3\log_2 x$	**M1**	Use $u_4 = a + 3d$	Allow missing / incorrect / inconsistent log bases. Starting with $\log_2 27 + \log_2 x^3$ is M0M0. Starting with $\log_2 27$ x $3\log_2 x$ is M0 (but can get M1 below). Starting with $\log_2 27 + \log_2 x + \log_2 x + \log_2 x$ can get full credit.
			$= \log_2 27 + \log_2 x^3$	**M1**	Use $b \log a = \log a^b$ on $3\log_2 x$	u_4 must still be shown as two terms. Could get M1 if using $a + 4d$. Could get M1 for $\log_2 27$ x $3\log_2 x = \log_2 27$ x $\log_2 x^3$ or for $\log_2 27$ x $3\log_2 x = \log_2 27 + \log_2 x^3$. Allow missing / incorrect / inconsistent log bases.
			$= \log_2(27x^3)$ **AG**	**A1** **[3]**	Show $\log_2(27x^3)$ convincingly	Can go straight from $\log_2 27 + \log_2 x^3$ to final answer. CWO, including using base 2 throughout. **SR – finding consecutive terms (each step must be explicit)** **B1** for $u_2 = \log_2 27 + \log_2 x = \log_2 27x$ **B1** for $u_3 = \log_2 27x + \log_2 x = \log_2 27x^2$ **B1** for $u_4 = \log_2 27x^2 + \log_2 x = \log_2 27x^3$
9	(a)	(ii)	$27x^3 = 2^6$	B1*	State correct equation no longer involving $\log_2 x$	Equation could still involve constant terms such as $\log_2 27$ or $\log_2 3$. Allow truncated or rounded decimals.
			$x = {}^4/_3$	B1d* **[2]**	Obtain ${}^4/_3$	Must be ${}^4/_3$, $1{}^1/_3$ or an exact recurring decimal only (not 1.333….). A0 if cube root still present. Working must be exact, so sight of decimals in method used is B0, even if final answer is exact. Answer only gets full credit.

Question			Answer	Marks	Guidance	
9	(b)	(i)	½ < y < 2	**M1**	Identify at least one of ½ and 2 as end-points	Only one end-point required. Ignore if additional incorrect end-point also given. Ignore any signs used.
				A1	Obtain ½ < y < 2	Not two separate inequalities, unless linked by 'and'. A0 for ½ ≤ y ≤ 2.
				[2]		
9	(b)	(ii)	$\frac{\log_2 27}{1-\log_2 y}=3$	B1	State $\frac{\log_2 27}{1-\log_2 y}=3$	Allow B1 if no base stated, but B0 if incorrect base. Must be equated to 3 for B1.
			$\log_2 27 = 3 - 3\log_2 y$ $\log_2 27 = 3 - \log_2 y^3$ $\log_2(27y^3) = 3$	M1*	Attempt to rearrange equation to $\log_2 f(y) = k$	Must be using $\frac{\log_2 27}{\pm 1 \pm \log_2 y}$ (but allow for no bases). Allow at most 2 manipulation errors (eg +/– or x/÷ muddles, or slips when expanding brackets) but M0 if other errors (eg incorrect use of logs).
			$27y^3 = 8$	M1d*	Use $f(y) = 2^k$ as inverse of $\log_2 f(y) = k$	Must have first been arranged to $\log_2 f(y) = k$. No need to go any further than stating their $f(y) = 2^k$.
				A1*	Obtain correct exact equation no longer involving $\log_2 y$	Equation could still involve constant terms such as $\log_2 27$ or $\log_2 3$. Sight of decimals used is A0, even if answer is exact.
			$y^3 = {}^8/_{27}$ $y = {}^2/_3$	A1d*	Obtain ${}^2/_3$	Allow equiv recurring decimal, but not 0.666… A0 if still cube root present.
				[5]		**SR** answer only is **B3** Correct $S_\infty = 3$, then answer with no further working is **B3.**